Math 242, Spring 1994, Exam 1

Use your own paper. SHOW your work. $\boxed{CIRCLE}$ your answer.

1. (15 points) State the Existence and Uniqueness Theorem for first order differential equations.

2. (25 points)
   (a) What does the Existence and Uniqueness Theorem tell you about the Initial Value Problem
   $$(1+x^2)y' = (2+y)^2 \quad y(0) = 0?$$
   (b) Solve the Initial Value Problem of part (a).
   (c) What does the Existence and Uniqueness Theorem tell you about the Initial Value Problem
   $$(1+x^2)y' = (2+y)^2 \quad y(0) = -2?$$
   (d) Solve the Initial Value Problem of part (c).

3. (15 points) Solve the Initial Value Problem $y' = x\sin x\,,\; y(0) = 0\,.$

4. (15 points) Solve the Initial Value Problem $xy' = 3y + x^4\cos x\,,\; y(2\pi) = 0\,.$

5. (15 points) A 400 gallon tank initially contains 100 gallons of brine containing 50 pounds of salt. Brine containing 1 pound of salt per gallon enters the tank at the rate of 5 gal./sec., and the mixed brine in the tank flows out at the rate the rate of 3 gal./sec.. How much salt will the tank contain at the moment it becomes full?

6. (15 points) The rate of decay of a radioactive substance is proportional to the amount of the substance present. Two grams of a radioactive substance are left to decay. It is observed that after a year 1.4 grams are left. What will the amount be after a total time of 10 years?