Math 242, 1993, Exam 1

There are 5 problems. Each problem is worth 20 points. Use your own paper. SHOW your work. $\boxed{CIRCLE}$ your answer. CHECK your answers.

1. Solve the Initial Value Problem

$$y'' + 6x^2 = 0 \quad y(1) = 2 \quad \text{and} \quad y'(1) = 3.$$

2. Solve $xy' + 3y = 3x^{-3/2}$.

3. Solve $xy' = y + 2\sqrt{xy}$.

4. Consider the Initial Value Problem

$$(*) \qquad (1 + x^2)y' = (1 + y)^2 \quad y(a) = b.$$

a. For which values of a and b does the Existence and Uniqueness Theorem guarantee that (*) has a unique solution.
b. Solve (*) for $a = b = 0$.
c. Solve (*) for $a = 0$ and $b = -1$.

5. A tank contains 1000 liters of a solution consisting of 100 kg of salt dissolved in water. Pure water is pumped into the tank at the rate of 5 liters/sec and the mixture — kept uniform by stirring — is pumped out at the same rate. How long will it be until only 10 kg of salt remain in the tank?