Quiz 5, October 25, 2016

Find the general solution of $y'' + 6y' + 9y = 0$.

Answer: We try $y = e^{rx}$. We solve the characteristic polynomial $r^2 + 6r + 9 = 0$. This polynomial is $(r+3)^2 = 0$. So, $r = -3$ is a root of multiplicity 2. The corresponding solutions of the Differential Equation are $y = e^{-3x}$ and $y = xe^{-3x}$. The general solution of the Differential Equation is

$$\boxed{y = c_1 e^{-3x} + c_2 x e^{-3x}.}$$

Check: We compute

$$y' = -3c_1 e^{-3x} + c_2(-3xe^{-3x} + e^{-3x})$$

and

$$y'' = 9c_1 e^{-3x} + c_2(9xe^{-3x} - 3e^{-3x} - 3e^{-3x}) = 9c_1 e^{-3x} + c_2(9xe^{-3x} - 6e^{-3x}).$$

Plug y into the DE to obtain

$$9c_1 e^{-3x} + c_2(9xe^{-3x} - 6e^{-3x}) + 6[-3c_1 e^{-3x} + c_2(-3xe^{-3x} + e^{-3x})] + 9[c_1 e^{-3x} + c_2 x e^{-3x}]$$
$$= c_1 e^{-3x}[9 - 18 + 9] + c_2 e^{-3x}[x(9 - 18 + 9) + (-6 + 6)] = 0.\checkmark$$