Please PRINT your name ______________________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

The quiz is worth 5 points. The solutions will be posted on my website later today.

E-mail your solution to

kustin@math.sc.edu

Quiz 9, Monday, April 5, 2021

Find the solution of the Initial Value Problem $y'' + 9y = \sin 2x$, $y(0) = 1$, $y'(0) = 0$.

Answer. Of course you know that the general solution of $y'' + 9y = 0$ is $y = c_1 \cos 3x + c_2 \sin 3x$. Also, it is easy to see that $y_{\text{particular}} = \frac{1}{5}\sin 2x$ is a particular solution of the given DE. It follows that the general solution of the DE $y'' + 9y = \sin 2x$ is $y = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{5}\sin 2x$. We must find c_1 and c_2 so that the Initial Conditions $y(0) = 1$ and $y'(0) = 0$ are also satisfied. We compute $y' = -3c_1 \sin 3x + 3c_2 \cos 3x + \frac{2}{5}\cos 2x$. Plug $x = 0$ into y and y' to obtain:

$$1 = y(0) = c_1 \quad \text{and} \quad 0 = y'(0) = 3c_2 + \frac{2}{5}.$$

We conclude that $c_1 = 1$ and $c_2 = -\frac{2}{15}$. Thus the answer is

$$\boxed{y = \cos 3x - \frac{2}{15}\sin 3x + \frac{1}{5}\sin 2x.}$$

Check. We take derivatives of $y = \cos 3x - \frac{2}{15}\sin 3x + \frac{1}{5}\sin 2x$ to obtain $y' = -3\sin 3x - \frac{2}{5}\cos 3x + \frac{2}{5}\cos 2x$ and $y'' = -9\cos 3x + \frac{6}{5}\sin 3x - \frac{4}{5}\sin 2x$. It is clear that $y'' + 9y = 3\sin 2x$. We plug 0 in for x to see that $y(0) = 1$ and $y'(0) = -\frac{2}{5} + \frac{2}{5} = 0$. ✓