

Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

The quiz is worth 5 points. The solutions will be posted on my website later today.

E-mail your solution to
kustin@math.sc.edu

Quiz 8, Monday, March 29, 2021

Solve the Initial Value Problem

$$x'' + 10x' + 125x = 0, \quad x(0) = 6, \quad x'(0) = 50.$$

If possible, put your answer in the form $x(t) = C_1 e^{-\beta t} \cos(\omega_1 t - \alpha_1)$.

Answer. We try $x = e^{rt}$. We immediately are faced with solving the characteristic equation:

$$r^2 + 10r + 125 = 0.$$

So, $r = \frac{-10 \pm \sqrt{100 - 500}}{2} = \frac{-10 \pm 20i}{2} = -5 \pm 10i$. So $x = c_1 e^{-5t} \cos 10t + c_2 e^{-5t} \sin 10t$ is the general solution of the differential equation. We now plug in the Initial Conditions to determine the values of c_1 and c_2 . We calculate

$$x' = e^{-5t} (-10c_1 \sin 10t - 5c_1 \cos 10t + 10c_2 \cos 10t - 5c_2 \sin 10t).$$

We have $6 = x(0) = c_1$ and $50 = x'(0) = -5c_1 + 10c_2$; so, $c_1 = 6$ and $c_2 = 8$. The solution is

$$x(t) = e^{-5t} (6 \cos 10t + 8 \sin 10t) = 10e^{-5t} \left(\frac{6}{10} \cos 10t + \frac{8}{10} \sin 10t \right).$$

We have arranged the numbers so that $(\frac{6}{10}, \frac{8}{10})$ is a point on the unit circle. Let $\alpha = \arccos(\frac{6}{10})$. It follows that $\sin \alpha = \frac{8}{10}$. We have

$$x(t) = 10e^{-5t} (\cos \alpha \cos 10t + \sin \alpha \sin 10t) = 10e^{-5t} \cos(\alpha - 10t) = 10e^{-5t} \cos(10t - \alpha).$$

Our answer is

$$x(t) = 10e^{-5t} \cos(10t - \alpha), \text{ where } \alpha = \arccos\left(\frac{3}{5}\right).$$