PRINT Your Name:________________________________

Quiz 8, Fall, 2012 – October 30

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. $\boxed{\text{BOX}}$ your answer.

1. Find the general solution of $6y^{(4)}+5y^{(3)}+25y''+20y'+4y=0$. You might find it helpful to know that $y=\cos 2x$ is a solution of the Differential Equation.

Answer. The hint tells us that $2i$ and $-2i$ are roots of the characteristic polynomial

$$6r^4+5r^3+25r^2+20r+4.$$

It follows that $(r-2i)(r+2i)=r^2+4$ is a factor of the characteristic polynomial. The other factor is $6r^2+5r+1$ and this may be found by performing long division:

$$6r^4+5r^3+25r^2+20r+4=(r^2+4)(6r^2+5r+1)=(r^2+4)(2r+1)(3r+1).$$

The roots of the characteristic polynomial are $\pm 2i$, $-1/2$, and $-1/3$. The general solution of the Differential equation is

$$\boxed{y=c_1\cos 2x+c_2\sin 2x+c_3e^{-x/2}+c_4e^{-x/3}.}$$

Check We take derivatives of our proposed answer:

$$\begin{aligned}
y' &= -2c_1\sin 2x+2c_2\cos 2x-(1/2)c_3e^{-x/2}-(1/3)c_4e^{-x/3}\\
y'' &= -4c_1\cos 2x-4c_2\sin 2x+(1/4)c_3e^{-x/2}+(1/9)c_4e^{-x/3}\\
y''' &= 8c_1\sin 2x-8c_2\cos 2x-(1/8)c_3e^{-x/2}-(1/27)c_4e^{-x/3}\\
y'''' &= 16c_1\cos 2x+16c_2\sin 2x+(1/16)c_3e^{-x/2}+(1/81)c_4e^{-x/3}
\end{aligned}$$

Plug our proposed solution back into the DE

$$\begin{aligned}
&6(16c_1\cos 2x+16c_2\sin 2x+(1/16)c_3e^{-x/2}+(1/81)c_4e^{-x/3})\\
&+5(8c_1\sin 2x-8c_2\cos 2x-(1/8)c_3e^{-x/2}-(1/27)c_4e^{-x/3})\\
&+25(-4c_1\cos 2x-4c_2\sin 2x+(1/4)c_3e^{-x/2}+(1/9)c_4e^{-x/3})\\
&+20(-2c_1\sin 2x+2c_2\cos 2x-(1/2)c_3e^{-x/2}-(1/3)c_4e^{-x/3})\\
&+4(c_1\cos 2x+c_2\sin 2x+c_3e^{-x/2}+c_4e^{-x/3})
\end{aligned}$$

$$=\begin{cases}
c_1[(96-100+4)\cos 2x+(40-40)\sin 2x]\\
+c_2[(-40+40)\cos 2x+(96-100+4)\sin 2x]\\
+c_3(\frac{6-10+100-160+64}{16})e^{-x/2}\\
+c_4(\frac{6-15+225-540+324}{81})e^{-x/3}
\end{cases}$$

$$=0.\ \checkmark$$