

Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 6, March 27, 2024

Solve the Initial Problem

$$\frac{1}{2}x'' + 3x' + 4x = 0, \quad x(0) = 2, \quad x'(0) = 0.$$

Solution. We try $x = e^{rt}$. We must study the characteristic polynomial

$$\frac{1}{2}r^2 + 3r + 4 = 0.$$

Multiply both sides by 2 and facotr:

$$r^2 + 6r + 8 = 0$$

$$(r + 2)(r + 4) = 0$$

So $r = -2$ and $r = -4$ are the roots of the characteristic polynomial. The general solution of the Differential Equation is

$$x = c_1e^{-2x} + c_2e^{-4x}.$$

We use the initial conditions to evaluate the constants. We compute

$$x = c_1e^{-2x} + c_2e^{-4x}$$

$$x' = -2c_1e^{-2x} - 4c_2e^{-4x}$$

It follows that

$$2 = c_1 + c_2$$

$$0 = -2c_1 - 4c_2$$

Replace Equation 2 with Equation 2 plus 2 times Equation 1:

$$2 = c_1 + c_2$$

$$4 = -2c_2$$

We conclude that $c_2 = -2$ and $c_1 = 4$. Thus,

$$\boxed{x = 4e^{-2t} - 2e^{-4t}}$$

Check. We plug

$$x = 4e^{-2t} - 2e^{-4t}$$

$$x' = -8e^{-2t} + 8e^{-4t}$$

$$x'' = 16e^{-2t} - 32e^{-4t}$$

into $\frac{1}{2}x'' + 3x' + 4x$ and obtain

$$\begin{aligned} & (1/2)(16e^{-2t} - 32e^{-4t}) \\ & + 3(-8e^{-2t} + 8e^{-4t}) \\ & + 4(4e^{-2t} - 2e^{-4t}) \end{aligned}$$

$$= e^{-2t}(8 - 24 + 16) + e^{-4t}(-16 + 24 - 8) = 0 \checkmark.$$

We also see that $x(0) = 4 - 2 = 2 \checkmark$ and $x'(0) = -8 + 8 = 0$.

Our proposed solution does everything it is supposed to do. It is correct.