Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

The quiz is worth 5 points. The solutions will be posted on my website later today.

E-mail your solution to

kustin@math.sc.edu

Quiz 6, Monday, March 8, 2021

Find the explicit solution for the Initial Value Problem

$$\frac{dx}{dt} = x^2 - 5x + 4 \quad x(0) = x_0.$$

ANSWER: Observe that $x^2 - 5x + 4 = (x-4)(x-1)$. Separate the variables and integrate:

$$\int \frac{dx}{(x-4)(x-1)} = \int dt.$$

Observe that

$$\frac{1}{(x-4)(x-1)} = \frac{1}{3}\left(\frac{1}{x-4} - \frac{1}{x-1}\right).$$

We integrate

$$\frac{1}{3}\int \left(\frac{1}{x-4} - \frac{1}{x-1}\right) dx = t + C$$

$$\frac{1}{3}(\ln|x-4| - \ln|x-1|) = t + C$$

$$\ln\left|\frac{x-4}{x-1}\right| = 3t + 3C$$

$$\left|\frac{x-4}{x-1}\right| = e^{3C}e^{3t}$$

$$\frac{x-4}{x-1} = \pm e^{3C}e^{3t}.$$

Let $K = \pm e^{3C}$.

$$(*) \quad \frac{x-4}{x-1} = Ke^{3t}.$$

Plug in $t = 0$ to learn that $\frac{x_0-4}{x_0-1} = K$. Multiply both sides of $(*)$ by $x-1$ to see that

$$x - 4 = Ke^{3t}(x-1)$$

Subtract $Ke^t x$ from both sides and add 4 to both sides:

$$x(1 - Ke^{3t}) = 4 - Ke^{3t}.$$

$$x = \frac{4 - Ke^{3t}}{1 - Ke^{3t}}$$

$$x = \frac{4 - \frac{x_0-4}{x_0-1}e^{3t}}{1 - \frac{x_0-4}{x_0-1}e^{3t}}$$

$$\boxed{x = \frac{4(x_0-1) - (x_0-4)e^{3t}}{(x_0-1) - (x_0-4)e^{3t}}}$$