Quiz 6, March 21, 2017, 11:40 class

Find a particular solution of $y'' - y' - 6y = 2\sin 3x$.

<u>Answer:</u> We look for a solution of the form $y = A\sin 3x + B\cos 3x$. We compute:

$$y' = 3A\cos 3x - 3B\sin 3x$$
$$y'' = -9A\sin 3x - 9B\cos 3x$$

Plug our candidate into the DE to obtain:

$$(-9A\sin 3x - 9B\cos 3x) - (3A\cos 3x - 3B\sin 3x) - 6(A\sin 3x + B\cos 3x) = 2\sin 3x.$$

Equate the coefficients of $\sin 3x$ and the coefficients of $\cos 3x$. We hope to solve

$$-9A + 3B - 6A = 2 \quad \text{and} \quad -9B - 3A - 6B = 0$$

We hope to solve

$$\begin{cases} -15A + 3B = 2 \\ -3A - 15B = 0 \end{cases}$$

Divide E2 by -3:

$$\begin{cases} -15A + 3B = 2 \\ A + 5B = 0 \end{cases}$$

$E1 \to E1 + 15E2$

$$\begin{cases} +78B = 2 \\ A + 5B = 0 \end{cases}$$

$B = 1/39, A = -5/39$. We have calculated that $\boxed{y = -5/39\sin 3x + 1/39\cos 3x}$ is a solution of the differential equation.

<u>Check:</u> Plug

$$\begin{cases} y = (1/39)(-5\sin 3x + 1\cos 3x) \\ y' = (1/39)(-3\sin 3x - 15\cos 3x) \\ y'' = (1/39)(+45\sin 3x - 9\cos 3x) \end{cases}$$

into the DE to obtain

$$(1/39)((+45\sin 3x - 9\cos 3x) - (-3\sin 3x - 15\cos 3x) - 6(-5\sin 3x + 1\cos 3x)$$

$$(1/39)(78\sin 3x + 0\cos 3x) = 3\sin 3x. \checkmark$$