

Please PRINT your name \_\_\_\_\_

**No calculators, cell phones, computers, notes, etc.**

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you.

The quiz is worth 5 points. The solutions will be posted on my website later today.

**Quiz 5, March 20, 2024**

Find the general solution of  $y'' + 6y' + 9y = 0$ .

**Solution.** We try  $y = e^{rx}$ . We plug  $y, y' = re^{rx}$  and  $y'' = r^2e^{rx}$  into the Differential Equation. We want

$$r^2e^{rx} + 6re^{rx} + 9e^{rx} = 0.$$

We want  $e^{rx}(r^2 + 6r + 9) = 0$ . If a product is zero, one of the factors must be zero. The function  $e^{rx}$  is never zero; so we want  $r^2 + 6r + 9 = 0$ . We want  $(r + 3)^2 = 0$ . It follows that  $y = e^{-3x}$  and  $y = xe^{-3x}$  are solutions of the given linear homogeneous Differential Equation with constant coefficients. The general solution of  $y'' + 6y' + 9y = 0$  is  $y = c_1e^{-3x} + c_2xe^{-3x}$ .

**Check.** We plug

$$\begin{aligned}y &= c_1e^{-3x} + c_2xe^{-3x} \\y' &= -3c_1e^{-3x} + c_2e^{-3x} - 3c_2xe^{-3x} \\&= (-3c_1 + c_2)e^{-3x} - 3c_2xe^{-3x} \\y'' &= -3(-3c_1 + c_2)e^{-3x} - 3c_2e^{-3x} + 9c_2xe^{-3x} \\&= (9c_1 - 6c_2)e^{-3x} + 9c_2xe^{-3x}\end{aligned}$$

into  $y'' + 6y' + 9y$  and obtain

$$\begin{aligned}&\begin{cases} (9c_1 - 6c_2)e^{-3x} + 9c_2xe^{-3x} \\ +6(-3c_1 + c_2)e^{-3x} - 3c_2xe^{-3x} \\ +9(c_1e^{-3x} + c_2xe^{-3x}) \end{cases} \\&= [(9 - 18 + 9)c_1 + (-6 + 6)c_2]e^{-3x} + (9 - 18 + 9)c_2xe^{-3x} = 0. \checkmark\end{aligned}$$