Please PRINT your name ______________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

The quiz is worth 5 points. The solutions will be posted on my website later today.

E-mail your solution to

`kustin@math.sc.edu`

Quiz 5, Monday, March 1, 2021

Let $P(t)$ represent the number of alligators in a certain park at time t. Suppose further that $P(t)$ satisfies the Differential Equation

$$\frac{dP}{dt} = \frac{B_0}{P_0^2}P^2 - \frac{D_0}{P_0}P,$$

where P_0 is the alligator population at time zero, B_0 is the birth rate at time zero, and D_0 is the death rate at time zero. The solution of the Differential Equation is

$$P(t) = \frac{M}{1 - \frac{P_0 - M}{P_0} e^{\frac{D_0 t}{P_0}}},$$

where $M = \frac{D_0 P_0}{B_0}$. **Use this value of $P(t)$. I do not expect you to solve the Differential equation or even to verify that the given solution is correct.**

Suppose that $P_0 = 100$ alligators, $B_0 = 10$ alligators per month, and $D_0 = 9$ alligators per month. When will the alligator population reach ten times M?

ANSWER: Observe that $P(t) = 10M$ when $10M = \frac{M}{1 - \frac{P_0 - M}{P_0} e^{\frac{D_0 t}{P_0}}}$. Multiply both sides by $\frac{1 - \frac{P_0 - M}{P_0} e^{\frac{D_0 t}{P_0}}}{10M}$ to obtain

$$1 - \frac{P_0 - M}{P_0} e^{\frac{D_0 t}{P_0}} = \frac{1}{10}.$$

$$\frac{9}{10} = \frac{P_0 - M}{P_0} e^{\frac{D_0 t}{P_0}}$$

$$\frac{9P_0}{10(P_0 - M)} = e^{\frac{D_0 t}{P_0}}$$

$$\frac{P_0}{D_0} \ln \frac{9P_0}{10(P_0 - M)} = t$$

$$\frac{100}{9} \ln \frac{900}{10(100 - 90)} = t$$

$$\frac{100}{9} \ln 9 = t$$

The population will reach $10M$ after $\frac{100}{9} \ln 9$ months.