PRINT Your Name:_____

Quiz for September 22, 2016

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. BOX your answer.

1. **The acceleration of a car is proportional to the difference between** 250 **ft/sec and the velocity of the car. If this car can accelerate from** 0 **to** 100 **ft/sec in** 10 **seconds, how long will it take for the car to accelerate from rest to** 150 **ft/sec?**

Let $v(t)$ be the velocity of the car (measured in ft/sec) at time t seconds. We are told that $\frac{dv}{dt} = k(250 - v)$. The initial condition is $v(0) = 0$. We are told that $v(10) = 100$. (This allows us to find k.) We are asked to find the time with $v(t) = 150$. We integrate $\int \frac{dv}{250-v} = \int k dt$ to see that

$$(*) \qquad -\ln(250 - v) = kt + C$$

The initial condition $v(0) = 0$ tells us that $-\ln 250 = C$. We plug in $v(10) = 100$ into (*) to see that $-\ln(250 - 100) = 10k - \ln 250$. It follows that

$$\ln 250 - \ln(150) = 10k$$

$$\ln \frac{250}{150} = 10k;$$

so, $\frac{\ln \frac{5}{3}}{10} = k$. We now find the time when $v(t) = 150$. Again, we use (*). We solve $-\ln(250 - 150) = kt + C$. We solve $-\ln(100) = (\frac{\ln \frac{5}{3}}{10})t - \ln 250$. We see that $t = \frac{\ln 250 - \ln 100}{\frac{\ln \frac{5}{3}}{10}} = 10\frac{\ln \frac{250}{100}}{\ln \frac{5}{3}} = \boxed{10\frac{\ln \frac{5}{2}}{\ln \frac{5}{3}} \text{sec}}$.