

PRINT Your Name: \_\_\_\_\_

**Quiz 5, Fall, 2012**

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. BOX your answer. **Check your answer**

Suppose that a motorboat is moving at 40 ft/sec when its motor suddenly quits, and that 10 seconds later the boat has slowed to 20 ft/sec. Assume that the resistance the boat encounters while coasting is proportional to its velocity. How far will the boat coast in all?

We are told  $dv/dt = -kv$ ,  $v(0) = 40$ ,  $v(10) = 20$ ,  $x(0) = 0$ . We want  $\lim_{t \rightarrow \infty} x(t)$ . We integrate  $dv/v = -kdt$  to learn  $\ln|v| = -kt + C$ . So,  $v = Ke^{-kt}$ . Plug in  $t = 0$  to learn that  $40 = K$ . Plug in  $t = 10$  to learn that  $(\ln 2)/10 = k$ . Integrate again to learn that  $x(t) = (K/(-k))e^{-kt} + C_1$ . Plug in  $t = 0$  to learn  $x(t) = (K/(-k))(e^{-kt} - 1)$ . Take  $\lim_{t \rightarrow \infty}$  to learn

$$\lim_{t \rightarrow \infty} x(t) = (K/k) = \boxed{400/(\ln 2) \text{ ft}}.$$