Please PRINT your name _____

The quiz is worth 5 points. Please make your work coherent, complete, and correct. Please $\boxed{\textit{CIRCLE}}$ your answer. Please **CHECK** your answer whenever possible.

The solution will be posted later today.

No Calculators, computers, smart phones, notes, etc.

Quiz 4, Febuary 22, 2018

A 120-gallon tank initially contains 90 lb of salt dissolved in 90 gal of water. Brine containing 2 lb/gal flows into the tank at the rate of 4 gal/min and the well-mixed mixture flows out of the tank at the rate of 3 gal/min. How much salt is in the tank when the tank is full?
Answer: Let $x(t)$ be the number of pounds of salt in the tank at time t. We are told that $x(0) = 90$. We are also told that

$$\frac{dx}{dt} = 2\frac{\text{lb}}{\text{gal}}4\frac{\text{gal}}{\text{min}} - \frac{x}{90+t}\frac{\text{lb}}{\text{gal}}3\frac{\text{gal}}{\text{min}}.$$

We are supposed to find $x(30)$.

The DE

$$\frac{dx}{dt} + \frac{3x}{90+t} = 8$$

is a first order linear DE. Multiply both sides by

$$\mu(t) = e^{\int \frac{3}{90+t}dt} = e^{3\ln(90+t)} = (90+t)^3$$

to obtain

$$(90+t)^3\frac{dx}{dt} + 3x(90+t)^2 = 8(90+t)^3$$

Observe that the left side is $\frac{d}{dt}((90+t)^3x)$. Integrate both sides to obtain

$$(90+t)^3x = 2(90+t)^4 + C.$$

Thus,

$$x = 2(90+t) + C(90+t)^{-3}.$$

Plug in $t = 0$ to see

$$90 = x(0) = 180 + C(90)^{-3}$$
$$-(90)^4 = C$$

$\boxed{\text{There are } x(30) = 2(90+30) - (90)^4(90+30)^{-3} \text{ pounds of salt in the tank when the tank is full.}}$