Quiz 4, February 16, 2017, 1:15 class

Solve $xy' = y + 2\sqrt{xy}$. Express your answer in the form $y = y(x)$. Please check your answer.
ANSWER: We use a homogeneous substitution. Let $v = \frac{y}{x}$. It follows that $xv = y$ and $x\frac{dv}{dx} + v = \frac{dy}{dx}$. The differential equation is

$$y' = \frac{y}{x} + 2\frac{\sqrt{xy}}{x}$$

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$$x\frac{dv}{dx} + v = v + 2\sqrt{v}$$

$$\int \frac{v^{-1/2}}{2} dv = \int \frac{1}{x} dx$$

$$\sqrt{v} = \ln|x| + C$$

$$v = (\ln|x| + C)^2$$

$$\boxed{y = x(\ln|x| + C)^2}$$

Check. We assume $0 < x$ and check $y = x(\ln(x) + C)^2$. When we plug our proposed answer into the left side of the DE we obtain

$$xy' = x[(2x(\ln(x) + C)\tfrac{1}{x} + (\ln(x) + C)^2] = (2x(\ln(x) + C) + x(\ln(x) + C)^2.$$

When we plug our proposed answer into the right side of the DE we obtain

$$y + 2\sqrt{xy} = x(\ln(x) + C)^2 + 2\sqrt{xx(\ln(x) + C)^2} = x(\ln(x) + C)^2 + 2x(\ln(x) + C)).$$

These agree. Our answer is correct.