PRINT Your Name:________________________________

Quiz for June 5, 2012

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. $\boxed{\text{BOX}}$ your answer.

1. **Solve** $yy'+x=\sqrt{x^2+y^2}$ **. Express your answer in the form** $y(x)$ **. Check your answer.**

This is a homogeneous problem. Divide both sides by x to write the problem as

$$\frac{y}{x}y'+1=\sqrt{1+\left(\frac{y}{x}\right)^2}.$$

Let $v=\frac{y}{x}$. In other words, $xv=y$. Take the derivative with respect to x to see that $xv'+v=y'$. We must solve

$$v(xv'+v)+1=\sqrt{1+v^2}.$$

We must solve

$$xv\frac{dv}{dx}=\sqrt{1+v^2}-v^2-1.$$

We must solve

$$v\frac{dv}{\sqrt{1+v^2}-v^2-1}=\frac{dx}{x}.$$

Integrate both sides. Let $w=1+v^2$. It follows that $dw=2vdv$. We must solve

$$\frac{1}{2}\int\frac{dw}{\sqrt{w}-w}=\ln|x|+C.$$

We have

$$\ln|x|+C=\frac{1}{2}\int\frac{dw}{\sqrt{w}(1-\sqrt{w})}.$$

Let $u=\sqrt{w}$. We have $du=\frac{1}{2}w^{-1/2}dw$.

We have

$$\ln|x|+C=\int\frac{du}{1-u}=-\ln|1-u|=-\ln|1-\sqrt{w}|=-\ln|1-\sqrt{1+v^2}|$$

$$=-\ln\left|1-\sqrt{1+\left(\frac{y}{x}\right)^2}\right|=-\ln\left|\frac{x-\sqrt{x^2+y^2}}{x}\right|=-\ln\left|x-\sqrt{x^2+y^2}\right|+\ln|x|.$$

Subtract $\ln|x|$ from both sides:

$$C = -\ln\left|x - \sqrt{x^2+y^2}\right|$$

or

$$\ln\left|x - \sqrt{x^2+y^2}\right| = -C.$$

Exponentiate. Let K be the new constant e^{-C}. We have

$$x - \sqrt{x^2+y^2} = K;$$

so $x - K = \sqrt{x^2+y^2}$ and $(x-K)^2 = x^2+y^2$ and $\boxed{\pm\sqrt{(x-K)^2-x^2} = y.}$

Check: We check $y = +\sqrt{(x-K)^2-x^2}$, with $K \le x$. We see that

$$y' = \frac{2(x-K)-2x}{2\sqrt{(x-K)^2-x^2}} = \frac{-K}{\sqrt{(x-K)^2-x^2}}.$$

So, $yy' + x = -K + x$. On the other hand,

$$\sqrt{x^2+y^2} = \sqrt{x^2+(x-K)^2-x^2} = \sqrt{(x-K)^2} = x - K.$$

Thus, $yy' + x = \sqrt{y^2+x^2}$ as required. ✓