

Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Math 242, Quiz 3, January 29, 2025

Use Euler's method to approximate $y(\frac{1}{2})$ where y is a solution of the Initial Value Problem

$$y' = -y \quad \text{and} \quad y(0) = 2.$$

Use two steps only; in other words, take $h = \frac{1}{4}$.

Solution. We put the picture on the next page. The number y_2 is our approximation of $y(\frac{1}{2})$.

We first find y_1 . The line segment from $(0, 2)$ to $(\frac{1}{4}, y_1)$ has slope equal to $m_1 = -2$:

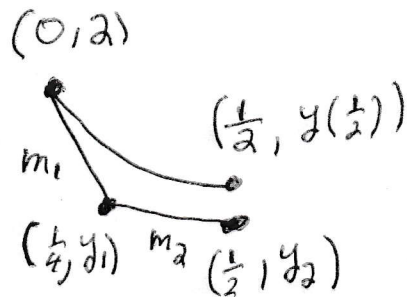
$$\frac{y_1 - 2}{\frac{1}{4}} = -2, \quad y_1 = 2 - \frac{1}{2} = \frac{3}{2}.$$

Now we find y_2 . The line segment from $(\frac{1}{4}, \frac{3}{2})$ to $(\frac{1}{2}, y_2)$ has slope equal to $m_2 = -y_1 = -\frac{3}{2}$:

$$\frac{y_2 - \frac{3}{2}}{\frac{1}{4}} = -\frac{3}{2}, \quad y_2 = \frac{3}{2} - \frac{3}{2} \cdot \frac{1}{4} = \frac{12}{8} - \frac{3}{8} = \frac{9}{8}.$$

Euler's Method, with $n = 2$, gives $\frac{9}{8}$ as the approximation of $y(\frac{1}{2})$.

Picture for 2.4 Number 1



The smooth curve is the real solution of the initial value problem $y' = -y$ $y(0) = 2$ (*)

The piece-wise linear curve is the Euler Method Approximation of the solution of (*) made using two steps.

m_1 is the slope of the line segment from $(0, 2)$ to $(\frac{1}{4}, y_1)$
 m_2 is the slope of the line segment from $(\frac{1}{4}, y_1)$ to $(\frac{1}{2}, y_2)$

We make $m_1 = f(0, 2)$ and $m_2 = f(\frac{1}{4}, y_1)$ where $f(x, y)$ is the right side of the Differential Equation in (*)
In other words $f(x, y) = -y$ $m_1 = f(0, 2) = -2$
 $m_2 = f(\frac{1}{4}, y_1) = -y_1$