Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 3, February 14, 2024

Solve $y' = y + y^3$.

ANSWER:

I will treat this problem as a Bernoulli Equation. (One could also solve the Differential Equation by separating the variables.)

Let $v = y^{1-3} = y^{-2}$. Observe that $\frac{dv}{dx} = -2y^{-3}\frac{dy}{dx}$. Multiply both sides of the original equation by $-2y^{-3}$ to obtain

$$-2y^{-3}y' = -2y^{-2} - 2$$
$$\frac{dv}{dx} = -2v - 2$$
$$(*) \quad \frac{dv}{dx} + 2v = -2.$$

This is a First Order Linear Differential Equation of the form $v' + P(x)v = Q(x)$, with $P(x) = 2$. We let $\mu(x) = e^{\int P(x)dx} = e^{\int 2dx} = e^{2x}$. Multiply both sides equation (*) by e^{2x} to obtain

$$(**) \quad e^{2x}\frac{dv}{dx} + 2e^{2x}v = -2e^{2x}.$$

The left side of (**) is equal to $\frac{d}{dx}(e^{2x}v)$; so, (**) is

$$\frac{d}{dx}(e^{2x}v) = -2e^{2x}.$$

Integrate both sides to obtain:

$$e^{2x}v = -e^{2x} + C.$$

Multiply both sides by e^{-2x} to obtain:

$$v = -1 + Ce^{-2x}$$
$$y^{-2} = -1 + Ce^{-2x}$$
$$\boxed{y = (-1 + Ce^{-2x})^{-1/2}}.$$

Check: We calculate

$$y' = (-1/2)(-1 + Ce^{-2x})^{-3/2}(-2Ce^{-2x}) = Ce^{-2x}(-1 + Ce^{-2x})^{-3/2}.$$

We also calculate

$$\begin{aligned} y+y^3 &= (-1+Ce^{-2x})^{-1/2}+(-1+Ce^{-2x})^{-3/2} \\ &= (-1+Ce^{-2x})^{-3/2}(-1+Ce^{-2x}+1) \\ &= Ce^{-2x}(-1+Ce^{-2x})^{-3/2}. \end{aligned}$$

We see that y' is equal to $y+y^3$. Our answer is correct.