Please PRINT your name _____

The quiz is worth 5 points. Please make your work coherent, complete, and correct. Please *CIRCLE* your answer. Please **CHECK** your answer whenever possible.

The solution will be posted later today.

No Calculators, computers, smart phones, notes, etc.

Quiz 3, Febuary 15, 2018

Solve $yy' + x = \sqrt{x^2+y^2}$. Express your answer in the form $y(x)$. **Check your answer.**
Answer: This is a homogeneous problem. Divide both sides by x to write the problem as

$$\frac{y}{x}y' + 1 = \sqrt{1+\left(\frac{y}{x}\right)^2}.$$

Let $v = \frac{y}{x}$. In other words, $xv = y$. Take the derivative with respect to x to see that $xv' + v = y'$.
We must solve

$$v(xv' + v) + 1 = \sqrt{1+v^2}.$$

We must solve

$$xv\frac{dv}{dx} = \sqrt{1+v^2} - v^2 - 1.$$

We must solve

$$v\frac{dv}{\sqrt{1+v^2} - v^2 - 1} = \frac{dx}{x}.$$

Integrate both sides. Let $w = 1+v^2$. It follows that $dw = 2vdv$. We must solve

$$\frac{1}{2}\int \frac{dw}{\sqrt{w} - w} = \ln|x| + C.$$

We have

$$\ln|x| + C = \frac{1}{2}\int \frac{dw}{\sqrt{w}(1-\sqrt{w})}.$$

Let $u = \sqrt{w}$. We have $du = \frac{1}{2}w^{-1/2}dw$.

We have

$$\ln|x| + C = \int \frac{du}{1-u} = -\ln|1-u| = -\ln|1-\sqrt{w}| = -\ln|1-\sqrt{1+v^2}|$$

$$= -\ln\left|1-\sqrt{1+\left(\frac{y}{x}\right)^2}\right| = -\ln\left|\frac{x-\sqrt{x^2+y^2}}{x}\right| = -\ln\left|x-\sqrt{x^2+y^2}\right| + \ln|x|.$$

Subtract $\ln|x|$ from both sides:

$$C = -\ln\left|x - \sqrt{x^2+y^2}\right|$$

or

$$\ln\left|x - \sqrt{x^2+y^2}\right| = -C.$$

Exponentiate. Let K be the new constant e^{-C}. We have

$$x - \sqrt{x^2+y^2} = K;$$

so $x - K = \sqrt{x^2+y^2}$ and $(x-K)^2 = x^2+y^2$ and $\boxed{\pm\sqrt{(x-K)^2 - x^2} = y.}$

Check: We check $y = +\sqrt{(x-K)^2-x^2}$, with $K \le x$. We see that

$$y' = \frac{2(x-K)-2x}{2\sqrt{(x-K)^2-x^2}} = \frac{-K}{\sqrt{(x-K)^2-x^2}}.$$

So, $yy' + x = -K + x$. On the other hand,

$$\sqrt{x^2+y^2} = \sqrt{x^2+(x-K)^2-x^2} = \sqrt{(x-K)^2} = x-K.$$

Thus, $yy' + x = \sqrt{y^2+x^2}$ as required. ✓