

Quiz 3, February 9, 2017, 1:15 class

Solve $y' = y + y^3$. Express your answer in the form $y = y(x)$. Please check your answer.

ANSWER: This is a Bernoulli equation. Let $v = y^{1-3} = y^{-2}$, Compute $\frac{dv}{dx} = -2y^{-3} \frac{dy}{dx}$.

Multiply both sides of the differential equation by $-2y^{-3}$ to obtain

$$-2y^{-3} \frac{dy}{dx} = -2y^{-2} - 2.$$

Replace $-2y^{-3} \frac{dy}{dx}$ with $\frac{dv}{dx}$ and y^{-2} with v . The DE has become

$$\frac{dv}{dx} + 2v = -2,$$

which is a first order linear DE. Multiply both sides by $\mu = e^{\int 2dx} = e^{2x}$ to obtain

$$e^{2x} \frac{dv}{dx} + 2e^{2x}v = -2e^{2x},$$

which is the same as

$$\frac{d}{dx}(e^{2x}v) = -2e^{2x}.$$

Apply $\int -dx$ to both sides

$$e^{2x}v = -e^{2x} + C.$$

$$v = -1 + Ce^{-2x}$$

$$y^{-2} = -1 + Ce^{-2x}.$$

$$y = \frac{\pm 1}{\sqrt{-1 + Ce^{-2x}}}.$$

Check. We check $y = \frac{1}{\sqrt{-1 + Ce^{-2x}}}$. We compute

$$y' = \left(\frac{-1}{2}\right)(-1 + Ce^{-2x})^{-3/2}(-2Ce^{-2x}) = \frac{Ce^{-2x}}{(-1 + Ce^{-2x})^{3/2}}.$$

We also compute

$$y + y^3 = \frac{1}{\sqrt{-1 + Ce^{-2x}}} + \frac{1}{(\sqrt{-1 + Ce^{-2x}})^3} = \frac{(-1 + Ce^{-2x}) + 1}{(\sqrt{-1 + Ce^{-2x}})^3} = \frac{Ce^{-2x}}{(\sqrt{-1 + Ce^{-2x}})^3}.$$

These expressions are equal. Our answer is correct.