

Quiz 3, February 9, 2017, 11:40 class

Solve $y' = (4x + y)^2$. Express your answer in the form $y = y(x)$. Please check your answer.

ANSWER: We make a linear substitution $v = 4x + y$. We calculate that $\frac{dv}{dx} = 4 + \frac{dy}{dx}$. Our problem now is

$$v' - 4 = v^2.$$

We separate the variables:

$$\begin{aligned}\int \frac{dv}{v^2 + 4} &= \int dx. \\ \frac{1}{2} \arctan \frac{v}{2} &= x + C \\ \arctan \frac{v}{2} &= 2x + 2C\end{aligned}$$

Let $K = 2C$.

$$\begin{aligned}\frac{v}{2} &= \tan(2x + K) \\ v &= 2 \tan(2x + K) \\ 4x + y &= 2 \tan(2x + K) \\ \boxed{y = 2 \tan(2x + K) - 4x}\end{aligned}$$

Check. We compute

$$y' = 4 \sec^2(2x + K) - 4.$$

We also compute

$$(4x + y)^2 = (4x + 2 \tan(2x + K) - 4x)^2 = (2 \tan(2x + K))^2 = 4 \tan^2(2x + K).$$

Of course, $\sin^2 \theta + \cos^2 \theta = 1$, so $\tan^2 \theta + 1 = \sec^2 \theta$. So,

$$(4x + y)^2 = 4(\sec^2(4x + y) - 1)$$

and this agrees with the value of y' . We have done the problem correctly.