Please PRINT your name _____

The quiz is worth 5 points. Please make your work coherent, complete, and correct. Please *CIRCLE* your answer. Please **CHECK** your answer whenever possible.

The solution will be posted later today.

No Calculators, computers, smart phones, notes, etc.

Quiz 2, January 30, 2018

A motor boat is moving at 40 feet per second when its motor suddenly quits and 10 seconds later the boat has slowed to 20 feet/second. The only force acting on the boat is resistance and resistance is proportional to velocity. How far will the boat coast in all?

Answer: Let $x(t)$ equal the position of the boat at time t, where t measures the amount of time since the motor quit. We are told

$$x'' = -kx', \quad x'(0) = 40, \quad x'(10) = 20, \quad \text{and} \quad x(0) = 0,$$

for some positive constant k. If you like, let $v = x'$. Separate the variables in $\frac{dv}{dt} = -kv$ and integrate $\int \frac{dv}{v} = \int -k dt$:

$$\ln|v| = -kt + C$$
$$|v| = e^{-kt+C}$$
$$v = Ke^{-kt}$$
$$x' = Ke^{-kt}$$

Plug in $t = 0$ to learn $K = 40$. So,

$$x' = 40e^{-kt}$$

Plug in $t = 10$ to learn

$$20 = 40e^{-10k}$$
$$\frac{1}{2} = e^{-10k}$$
$$-\ln 2 = -10k$$
$$\frac{\ln 2}{10} = k$$

Integrate with respect to t to see that

$$x = \frac{40}{-k}e^{-kt} + C_1$$

Plug in $t = 0$ to see that

$$0 = \frac{40}{-k} + C_1$$

so $C_1 = \frac{40}{k}$ and $x(t) = \frac{40}{k} - \frac{40}{-k}e^{-kt}$. We see that x' is never zero; but $\lim_{t\to\infty} x' = 0$. The total distance traveled by the boat is

$$\lim_{t\to\infty} x = \lim_{t\to\infty} \frac{40}{k} - \frac{40}{-k}e^{-kt} = \frac{40}{k} = \frac{40}{\frac{\ln 2}{10}} = \boxed{\frac{400}{\ln 2}\text{feet}}.$$