

Please PRINT your name \_\_\_\_\_

**No calculators, cell phones, computers, notes, etc.**

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

### Quiz 2, September 27, 2023

Solve the Initial Value Problem

$$y' + y = e^x \quad \text{and} \quad y(0) = 1.$$

Check that your answer is correct.

**ANSWER:** This is a first order linear problem of the form

$$y' + P(x)y = Q(x).$$

The trick is to multiply both sides of the equation by

$$\mu(x) = e^{\int P(x)dx}.$$

Our  $P(x)$  is equal to 1; so our

$$\mu(x) = e^{\int 1dx} = e^x.$$

We multiply both sides by  $\mu(x)$  to obtain

$$e^x y' + e^x y = e^{2x}.$$

Notice that the left side is equal to

$$\frac{d}{dx}(e^x y).$$

So we must solve

$$\frac{d}{dx}(e^x y) = e^{2x}.$$

We integrate both sides with respect to  $x$ . We obtain

$$e^x y = \frac{1}{2}e^{2x} + C.$$

We use the initial condition  $y(0) = 1$  to find  $C$ .

$$e^0(1) = \frac{1}{2}e^0 + C.$$

We see that  $C = \frac{1}{2}$ . The answer is

$$e^x y = \frac{1}{2}e^{2x} + \frac{1}{2}.$$

Divide both sides by  $e^x$ :

$$y = \frac{1}{2} \frac{(e^{2x} + 1)}{e^x}.$$

That is a fine answer. It might be prettier in the form

$$y = \frac{(e^x + e^{-x})}{2}.$$

**Check.**  $y(0) = \frac{1+1}{2} = 1 \checkmark$ .

Also,

$$y' + y = \frac{(e^x - e^{-x})}{2} + \frac{(e^x + e^{-x})}{2} = e^x \checkmark$$