Quiz 2, September 1, 2016

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** Express your work in a coherent and correct manner. $\boxed{\text{BOX}}$ your answer. Solve

$$y^3\frac{dy}{dx} = (y^4+1)\cos x.$$

Express your answer in the form $y = y(x)$. Check your answer.

ANSWER: Multiply both sides by dx; divide both sides by y^4+1; integrate both sides:

$$\int \frac{y^3}{y^4+1}dy = \int \cos x dx.$$

$$\frac{1}{4}\ln(y^4+1) = \sin x + C$$

$$\ln(y^4+1) = 4\sin x + 4C$$

$$y^4+1 = e^{4C}e^{4\sin x}$$

$$y^4 = e^{4C}e^{4\sin x} - 1$$

$$\boxed{y = \pm(Ke^{4\sin x}-1)^{1/4}},$$

where $K = e^{4C}$.

CHECK: We compute

$$\frac{dy}{dx} = \pm\frac{1}{4}(Ke^{4\sin x}-1)^{-3/4}Ke^{4\sin x}4\cos x.$$

Thus, the left side of the Differential Equation is equal to

$$y^3\frac{dy}{dx} = \left(\pm(Ke^{4\sin x}-1)^{3/4}\right)\left(\pm\frac{1}{4}(Ke^{4\sin x}-1)^{-3/4}Ke^{4\sin x}4\cos x\right) = Ke^{4\sin x}4\cos x.$$

On the other hand, the right side of the differential equation is

$$(y^4+1)\cos x = \left[\left(\pm(Ke^{4\sin x}-1)^{1/4}\right)^4+1\right]\cos x = \left[(Ke^{4\sin x}-1)+1\right]\cos x = Ke^{4\sin x}\cos x.$$

The two sides are equal. We have solved the Differential Equation.