PRINT Your Name:________________________________

Quiz 10, Fall, 2012

Each quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner.

Use the method of Laplace transforms to solve the Initial Value Problem

$$x'' + x = \cos 3t, \quad x(0) = 1,\ x'(0) = 0.$$

Answer. Let $X = \mathcal{L}(x)$. We have $\mathcal{L}(x') = s\mathcal{L}(x) - x(0) = sX - 1$, $\mathcal{L}(x'') = s\mathcal{L}(x') - x'(0) = s^2X - s$, and $\mathcal{L}(\cos 3t) = \frac{s}{s^2+9}$. Transform the given IVP to

$$s^2X - s + X = \frac{s}{s^2+9}.$$

We solve for X:

$$(s^2+1)X = \frac{s}{s^2+9} + s$$

$$X = \frac{s}{(s^2+1)(s^2+9)} + \frac{s}{s^2+1}$$

Apply the technique of partial fractions to get

$$X = \frac{1}{8}\left[\frac{1}{s^2+1} - \frac{1}{s^2+9}\right] + \frac{s}{s^2+1}$$

$$X = \frac{1}{8}\left[9\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]$$

Apply the inverse Laplace Transform:

$$x = \mathcal{L}^{-1}(X) = \mathcal{L}^{-1}\left(\frac{1}{8}\left[9\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]\right) = \frac{1}{8}\left[9\cos t - \cos 3t\right].$$

Our answer is

$$\boxed{x(t) = \frac{1}{8}\left[9\cos t - \cos 3t\right].}$$