PRINT Your Name:____________________________________

Quiz 1, Spring, 2013

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. $\boxed{\text{BOX}}$ your answer. The solution will be posted later today.

Suppose the velocity of a motorboat coasting in water satisfies the differential equation $\frac{dv}{dt} = kv^2$. The initial speed of the motorboat is $v(0) = 10$ meters per second (m/s), and v is decreasing at the rate 1 m/s^2 when $v = 5$ m/s. How long does it take for the velocity of the boat to decrease to 1 m/s?

ANSWER: We separate the variables and integrate to see that $\int \frac{dv}{v^2} = \int k dt$; so, $-1/v = kt + C$. Plug in $t = 0$ to learn that $-1/10 = C$. Let t_1 be the time when $v = 5$. We are told that at time t_1, we have $\frac{dv}{dt}(t_1) = -1$. Plug t_1 into $\frac{dv}{dt} = kv^2$ to learn:

$$-1 = \frac{dv}{dt}(t_1) = kv(t_1)^2 = k(25).$$

So, $k = -1/25$. Thus,

$$v = \frac{-1}{kt + C} = \frac{-1}{\frac{-1}{25}t + \frac{-1}{10}}.$$

Multiply top and bottom by -50 to get

$$v = \frac{50}{2t + 5}.$$

We find the time t_2, when $v(t_2) = 1$:

$$1 = \frac{50}{2t_2 + 5}$$

$$2t_2 + 5 = 50$$

$$2t_2 = 45$$

$$\boxed{t_2 = 22.5\text{seconds}}$$