

Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Math 242, Quiz 1, January 15, 2025

Check that $y_1 = \frac{1}{x^2}$ and $y_2 = \frac{\ln x}{x^2}$ are solutions of the Differential Equation $x^2y'' + 5xy' + 4y = 0$, where $y' = \frac{dy}{dx}$.

Solution. We plug

$$\begin{aligned}y_1 &= x^{-2} \\y_1' &= -2x^{-3} \\y_1'' &= 6x^{-4}\end{aligned}$$

into $x^2y'' + 5xy' + 4y$ and obtain

$$x^2(6x^{-4}) + 5x(-2x^{-3}) + 4x^{-2} = x^{-2}(6 - 10 + 4) = 0. \checkmark$$

We plug

$$\begin{aligned}y_2 &= \frac{\ln x}{x^2} \\y_2' &= \ln x(-2)x^{-3} + x^{-3} \\y_2'' &= \ln x(6x^{-4}) - 2x^{-4} - 3x^{-4} = 6x^{-4}\ln x - 5x^{-4}\end{aligned}$$

into $x^2y'' + 5xy' + 4y$ and obtain

$$\begin{aligned}x^2(6x^{-4}\ln x - 5x^{-4}) + 5x((-2)x^{-3}\ln x + x^{-3}) + 4x^{-2}\ln x \\= x^{-2}\ln x(6 - 10 + 4) + x^{-2}(-5 + 5) = 0 \checkmark\end{aligned}$$

We see that y_1 and y_2 both are solutions of the Differential Equation.