Quiz 1, January 17, 2017, 1:15 class

Suppose a population P of rodents satisfies the differential equation $\frac{dP}{dt} = kP^2$. Initially, there are $P(0) = 2$ rodents, and their number is increasing at the rate of $\frac{dP}{dt} = 1$ rodent per month when there are $P = 10$ rodents. How long will it take for this population to grow to a hundred rodents? To a thousand? What is happening here?

ANSWER: We can find k right away. On the one hand, $\frac{dP}{dt} = kP^2$. On the other hand, when $P = 10$, $\frac{dP}{dt} = 1$. So $1 = \frac{dP}{dt}|_{P=10} = k(10)^2$. We see that $\frac{1}{100} = k$. Now we solve the differential equation:
$\frac{dP}{dt} = \frac{1}{100}P^2$. Separate the variables $dP/P^2 = (1/100)dt$. Integrate both sides:
$-1/P = (1/100)t + C$. Plug in $t = 0$ to learn: $-1/2 = C$. We have found
$-1/P = (1/100)t - (1/2)$. Multiply both sides by -100 to get $100/P = -t + 50$ or

$$\boxed{\frac{100}{50-t} = P(t)}.$$

We find t with $P(t) = 100$. So, $\frac{100}{50-t} = 100$ or $1 = 50 - t$ that is $t = 49$.

$$\boxed{\text{The rodent population hits 100 at } t \text{ equal to 49 months.}}$$

We find t with $P(t) = 1000$. So, $\frac{100}{50-t} = 1000$ or $\frac{1}{10} = 50 - t$ that is $t = 50 - \frac{1}{10}$.

$$\boxed{\text{The rodent population hits 100 at } t \text{ equal to } 50 - \frac{1}{10} \text{ months.}}$$

We see that

$$\lim_{t \to 50^-} P(t) = \lim_{t \to \to 50^-} \frac{100}{50-t} = +\infty.$$

$$\boxed{\text{The rodent population heads to infinity as } t \text{ goes to 50 months.}}$$