PRINT Your Name:____________________________________

Quiz 1, Fall, 2012

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. $\boxed{\text{BOX}}$ your answer.

On planet Gzyx, a ball dropped from a height of 20 ft hits the ground in 2 seconds. If a ball is dropped from the top of a 200-ft-tall building on Gzyx, how long will it take to hit the ground? With what speed will it hit?

ANSWER: Let $x(t)$ be the height of the ball above the ground at time t. Measure t in seconds and x in feet. We assume that $x''(t) = -k$ for some positive constant k. For the first event, we have $x(0) = 20$, $x'(0) = 0$, and $x(2) = 0$. For the second event, we have $x(0) = 200$ and $x'(0) = 0$. We want to find t_1 with $x(t_1) = 0$. We also want to find $x'(t_1)$.

We first think about the first event. Integrate to learn $x'(t) = -kt + C_1$. Plug in $x'(0) = 0$ to learn that $C_1 = 0$. Integrate again to learn $x(t) = -kt^2/2 + C_2$. Plug in $x(0) = 20$ to learn $C_2 = 20$. So, $x(t) = -kt^2/2 + 20$. Plug in $x(2) = 0$ to learn $k = 10$.

Now turn to the second event. Integrate twice and evaluate the constants to learn that $x'(t) = -kt$ and $x(t) = -kt^2/2 + 200$; with $k = 10$; so, $x(t) = -5t^2 + 200$. Solve $0 = x(t_1) = -5t^2 + 200$ to learn that $t_1 = \sqrt{40} = 2\sqrt{10}$ and $x'(t_1) = -10t_1 = -20\sqrt{10}$.

It takes the second ball $2\sqrt{10}$ seconds to hit the ground. The ball is traveling downward at the speed $20\sqrt{10}$ feet per second when it hits the ground.