

PRINT Your Name: _____

Quiz 8 on March 8, 2012

Each quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner.

Solve the Differential Equation

$$(DE) \quad 6y^{(4)} + 5y^{(3)} + 25y'' + 20y' + 4y = 0.$$

One of the solutions of (DE) is $y = \cos 2x$.

Answer. We try $y = e^{rx}$. We immediately are faced with solving the characteristic equation:

$$(CE) \quad 6r^4 + 5r^3 + 25r^2 + 20r + 4 = 0.$$

The problem tells us that $y = \cos 2x$ is a solution of (DE); so $r = 2i$ and $r = -2i$ are solutions of (CE). In other words, $r^2 + 4$ is a factor of (CE). Long divide to see that the other factor is $6r^2 + 5r + 1 = 0$. That is,

$$6r^4 + 5r^3 + 25r^2 + 20r + 4 = (r^2 + 4)(6r^2 + 5r + 1) = (r^2 + 4)(3r + 1)(2r + 1).$$

The roots of (CE) are $2i$, $-2i$, $-1/3$, and $-1/2$. The general solution of (DE) is

$$y = c_1 \cos 2x + c_2 \sin 2x + c_3 e^{-x/3} + c_4 e^{-x/2}.$$