

PRINT Your Name: \_\_\_\_\_

**Two Quizzes for March 1, 2012**

Each quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner.

1. When the brakes are applied to a certain car, the acceleration of the car is  $-k \text{ m/s}^2$  for some positive constant  $k$ . Suppose that the car is traveling at the velocity  $v_0 \text{ m/s}$  when the brakes are first applied and that the brakes continue to be applied until the car stops.
  - (a) Find the distance that the car travels between the moment that the brakes are first applied and the moment when the car stops. (Your answer will be expressed in terms of  $k$  and  $v_0$ .)
  - (b) How does the stopping distance change if the initial velocity is changed to  $4v_0$ ?

**Answer.** (a) Let  $x(t)$  be the position of the car at time  $t$ . We take  $t = 0$  to be the moment that the brakes are applied. So  $v(0) = v_0$  and  $x(0) = 0$ . We are told  $x'' = -k$ . We integrate and plug in the points to see  $v(t) = -kt + v_0$  and  $x(t) = -kt^2/2 + v_0t$ . Let  $t_s$  be the time when the car stops. We have  $0 = v(t_s) = -kt_s + v_0$ . Thus,  $t_s = v_0/k$ . The distance traveled while the brakes were applied is

$$x(t_s) = x(v_0/k) = -k(v_0/k)^2/2 + v_0(v_0/k) = (v_0^2/k)(1 - 1/2) = \boxed{\frac{v_0^2}{2k}}.$$

(b) The stopping distance is multiplied by  $4^2$  if  $v_0$  is replaced by  $4v_0$ .

2. Find a particular solution for each of the following Differential Equations:

- (a)  $y'' + 2y = 4$
- (b)  $y'' + 2y = 6x$
- (c)  $y'' + 2y = 6x + 4$

**Answer.** (a)  $y = 2$  (b)  $y = 3x$  (c)  $y = 3x + 2$ .