PRINT Your Name:____________________________________

Quiz for May 30, 2012

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. $\boxed{\text{BOX}}$ your answer.

Solve $(1-x^2)\frac{dy}{dx} = 2y$. Express your answer in the form $y = y(x)$. Check your answer.

ANSWER: Separate the variables to get $\frac{dy}{y} = \frac{2}{1-x^2}dx$. Integrate both sides:

$$\int \frac{dy}{y} = \int \frac{2}{1-x^2}dx.$$

Use the method of partial fractions:

$$\int \frac{dy}{y} = \int \left(\frac{1}{1-x} + \frac{1}{1+x}\right) dx$$

$$\ln|y| = -\ln|1-x| + \ln|1+x| + C.$$

Exponentiate:

$$e^{\ln|y|} = e^{-\ln|1-x|+\ln|1+x|+C}$$

$$|y| = e^C \tfrac{|1+x|}{|1-x|}$$

$$y = \pm e^C \tfrac{1+x}{1-x}$$

$$\boxed{y = K\tfrac{1+x}{1-x}},$$

where $K = \pm e^C$.

Check. We take the derivative of the proposed answer

$$\frac{dy}{dx} = K\frac{(1-x)-(1+x)(-1)}{(1-x)^2} = K\frac{2}{(1-x)^2}.$$

Plug $\frac{dy}{dx}$ into the left side of the DE to get

$$K(1-x^2)\frac{dy}{dx} = K(1-x^2)\frac{2}{(1-x)^2} = K\frac{2(1+x)}{1-x} = 2y. \checkmark$$