Math 242, Exam 3 SOLUTION, Spring 2010

Write everything on the blank paper provided.

You should KEEP this piece of paper.

If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **4** problems.

SHOW your work. $\boxed{CIRCLE}$ your answer. Write **coherently**.

No Calculators or Cell phones.

I will post the solutions later today.

1. **(12 points) Find the general solution of** $y'' - 4y = 2e^{2x}$ **.**

We first solve the corresponding homogeneous equation:

$$\text{(hom1)} \qquad y'' - 4y = 0.$$

For this, we try $y = e^{rx}$. We see that $y = e^{rx}$ is a solution of (hom1) precisely when $r^2 - 4 = 0$ or $r = 2$ or $r = -2$. The general solution of (hom1) is $y = c_1 e^{2x} + c_2 e^{-2x}$. Now we look for a particular solution of the original DE. Normally, we would try $y = Ae^{2x}$; however e^{2x} is a solution of the corresponding homogeneous DE so it will not help us solve the non-homogeneous DE. Instead, we try $y = Axe^{2x}$. We compute $y' = 2Axe^{2x} + Ae^{2x}$ and $y'' = 4Axe^{2x} + 2Ae^{2x} + 2Ae^{2x}$. When we plug this candidate into the original DE we obtain:

$$(4Axe^{2x} + 4Ae^{2x}) - 4(Ae^{2x}) = 2e^{2x},$$

which is

$$4Ae^{2x} = 2e^{2x}.$$

We take $A = \frac{1}{2}$. So $y_p = \frac{1}{2}xe^{2x}$ is a particular solution of the original DE and the general solution of the DE is

$$\boxed{y = c_1 e^{2x} + c_2 e^{-2x} + \tfrac{1}{2}xe^{2x}.}$$

2. **(12 points) Find the general solution of** $3y'' + y' - 2y = 2\cos x$ **.**

We first solve the corresponding homogeneous equation:

$$3y'' + y' - 2y = 0. \qquad \text{(hom2)}$$

For this, we try $y = e^{rx}$. We see that $y = e^{rx}$ is a solution of (hom2) precisely when $3r^2 + r - 2 = 0$; that is, $(3r-2)(r+1) = 0$, or $r = 2/3, -1$. The general solution of (hom2) is $y = c_1 y^{(2/3)x} + c_2 e^{-x}$. Now we look for a particular solution of the original DE. We try $y = A\cos x + B\sin x$. We compute $y' = -A\sin x + B\cos x$ and $y'' = -A\cos x - B\sin x$. We plug this candidate into the original DE. We want

$$3(-A\cos x - B\sin x) + (-A\sin x + B\cos x) - 2(A\cos x + B\sin x) = 2\cos x.$$

We want

$$(-3A + B - 2A)\cos x + (-3B - A - 2B)\sin x = 2\cos x.$$

We want

$$-5A + B = 2 \quad \text{and} \quad -5B - A = 0.$$

We want

$$A = -5B \quad \text{and} \quad -5(-5B) + B = 2.$$

We want

$$B = \tfrac{1}{13} \quad \text{and} \quad A = \tfrac{-5}{13}.$$

So, $y_p = \frac{-5}{13}\cos x + \frac{1}{13}\sin x$ is a particular solution of the original DE and

$$\boxed{y = c_1 y^{(2/3)x} + c_2 e^{-x} + \tfrac{-5}{13}\cos x + \tfrac{1}{13}\sin x}$$

is the general solution of the original DE.

3. **(13 points) Solve the Initial Value Problem** $2y'' + 12y' + 50y = 0$ **,** $y(0) = 0$ **,** $y'(0) = -8$ **.**

Try $y(x) = e^{rx}$. We solve $2r^2 + 12r + 50 = 0$. We solve $2(r^2 + 6r + 25) = 0$. We have

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 100}}{2} = \frac{-6 \pm 8i}{2} = -3 \pm 4i,$$

and the general solution of the Differential Equation is

$$y(x) = e^{-3x}(c_1 \cos 4x + c_2 \sin 4x).$$

We compute

$$y'(x) = e^{-3x}(-4c_1 \sin 4x + 4c_2 \cos 4x) - 3e^{-3x}(c_1 \cos 4x + c_2 \sin 4x).$$

Plug the initial condition into our general solution to obtain:

$$0 = y(0) = c_1 \quad \text{and} \quad -8 = y'(0) = 4c_2 - 3c_1 = 4c_2.$$

So, $c_2 = -2$ and

$$\boxed{y(x) = -2e^{-3x} \sin 4x.}$$

4. **(13 points) Find the general solution of** $9y''' + 12y'' + 4y' = 0$ **.**

Try $y(x) = e^{rx}$. We solve $9r^3 + 12r^2 + 4r = 0$. We solve $r(3r+2)^2 = 0$. The general solution of the Differential Equation is

$$\boxed{y = c_1 + e^{\frac{-2x}{3}}(c_2 + c_3 x).}$$