Math 242, Exam 2, Spring 2012

Write everything on the blank paper provided.

You should KEEP this piece of paper.

If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **7** problems.

SHOW your work. $\boxed{CIRCLE}$ your answer. Write **coherently**.

No Calculators or Cell phones.

I will post the solutions later today.

1. (8 points) Solve $xy\frac{dy}{dx} = y^2 + x\sqrt{4x^2 + y^2}$. Express your answer in the form y is some function of x . **Check** your answer.

2. (7 points) Solve $x^2\frac{dy}{dx} + 2xy = 5y^4$. Express your answer in the form y is some function of x . **Check** your answer.

3. (7 points) State the Existence and Uniqueness Theorem for first order differential equations.

4. (7 points) A tank contains 1000 liters (L) of a solution consisting of 100 kg of salt dissolved in water. A salt water solution which contains 2 kg of salt in each liter of solution is pumped into the tank at the rate of 5 L/s, and the mixture — kept uniform by stirring — is pumped out at the same rate. How long will it be until there are 500 kg of salt in the tank?

5. (7 points) When the brakes are applied to a certain car, the acceleration of the car is $-k\,\mathrm{m/s}^2$ for some positive constant k . Suppose that the car is traveling at the velocity $v_0\,\mathrm{m/s}$ when the brakes are first applied and that the brakes continue to be applied until the car stops.
   (a) Find the distance that the car travels between the moment that the brakes are first applied and the moment when the car stops. (Your answer will be expressed in terms of k and v_0 .)
   (b) How does the stopping distance change if the initial velocity is changed to $4v_0$?

PLEASE TURN OVER.

6. (7 points) Consider the Initial Value Problem $\frac{dx}{dt} = (x-1)(x-3)$, $x(0) = x_0$.
   (a) Solve the Initial Value Problem.
   (b) Draw some of the solutions.
   (c) Which choices for x_0 cause x to go to infinity at some finite time?
   (d) Which choices for x_0 cause x to go toward a finite constant as t goes to infinity.

7. (7 points) Consider the initial value problem $\frac{dy}{dx} = x + y^2$, $y(1) = 2$. Use Euler's method to approximate $y(3/2)$. Use two steps, each of size $1/4$.