Math 242, Exam 2, Fall 2012

You should KEEP this piece of paper. Write everything on the blank paper provided. If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible.

Nothing may be on your desk except things that came from me. In particular, **no Calculators or Cell phones** may be on your desk.

Your work must be coherent and correct. **I expect you to solve initial value problems. Unexplained, random formulas will not be accepted!**

The solutions will be posted later today.

1. (8 points) Suppose that a body moves through a resisting medium with resistance proportional to its velocity $v(t)$, so that $\frac{dv}{dt} = -kv$, for some positive constant k. Let $x(t)$ be the position of the object at time t. Let $v(0) = v_0$ and $x(0) = x_0$.
   (a) Find the velocity of the object at time t.
   (a) Find the position of the object at time t.
   (c) Find $\lim_{t \to \infty} x(t)$.

2. (7 points) Consider the initial value problem $\frac{dy}{dx} = 2x + y^3$, $y(1) = 2$. Use Euler's method to approximate $y(3/2)$. Use two steps, each of size $1/4$.

3. (7 points) Consider the Initial Value Problem $\frac{dx}{dt} = 9 - x^2$, $x(0) = x_0$.
   (a) Sketch the solution of this IVP for various values of x_0.
   (b) Solve the Initial Value Problem. Write your answer in the form $x = x(t)$.

Is your answer to (b) consistent with your answer to (a)?

4. (7 points) Solve $(x + y)\frac{dy}{dx} = 1$. (Your solution may be given as an implicit function. (In other words, you are not required to write your solution in the form $y = y(x)$.) Check your answer.

5. (7 points) Solve $x(x + y)\frac{dy}{dx} + y(3x + y) = 0$. Write your solution in the form $y = y(x)$. Check your answer.

Please turn over.

6. (7 points) A pitcher of buttermilk initially at $40° $ C is to be cooled by setting it on the front porch, where the temperature is $15°$ C. Suppose that the temperature of the buttermilk has dropped to $30°$ C after 25 minutes. When will the temperature of the buttermilk reach $20°$ C? (Recall that Newton's Law of Cooling states that the rate at which an object cools is proportional to the difference between the temperature of the object and the temperature of the surrounding medium.)

7. (7 points) Suppose the velocity of a motorboat coasting in water satisfies the differential equation $\frac{dv}{dt} = kv^2$. The initial speed of the motorboat is $v(0) = 10$ meters per second (m/s), and v is decreasing at the rate 1 m/s ^{2} when $v = 5$ m/s. How long does it take for the velocity of the boat to decrease to 1 m/s?