Math 242, Exam 1, Spring 2013

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones.

The solutions will be posted later today.

1. (8 points) Find all constants r for which $y = e^{rx}$ is a solution of $2y'' - 5y' + 3y = 0$. **Your work must be coherent and meaningful.**

2. (7 points) On the planet Gzyx, a ball dropped from a height of 80 ft hits the ground in 5 seconds. If a ball is dropped from the top of a 200-ft-tall building on Gzyx, how long will it take to hit the ground? With what speed will it hit? **I expect you to solve initial value problems. Unexplained, random formulas will not be accepted!** (Recall that Newton's Law of Motion states that if $F(t)$ is the force acting on an object moving in a straight line at time t, m is the mass of the object, and $a(t)$ is the acceleration of the object at time t, then $F = ma$. The only force acting on this ball on planet Gzyx is the force of gravity and this force is constant.) **Your work must be coherent and meaningful.**

3. (7 points) When the brakes are applied to a certain car, the acceleration of the car is $-k\frac{\mathrm{m}}{\mathrm{s}^2}$ for some positive constant k. Suppose that the car is traveling at the velocity $v_0\frac{\mathrm{m}}{\mathrm{s}}$ when the brakes are first applied and that the brakes continue to be applied until the car stops. **I expect you to solve initial value problems. Unexplained, random formulas will not be accepted! Your work must be coherent and meaningful.**
   (a) Find the distance that the car travels between the moment that the brakes are first applied and the moment when the car stops. (Your answer will be expressed in terms of k and v_0.)
   (b) How does the stopping distance change if the initial velocity is changed to $6v_0$?

Please turn over.

4. (7 points) Solve $\frac{dy}{dx} = (4x + y)^2$. Express your answer in the form $y = y(x)$. Check your answer. **Your work must be coherent and meaningful.**

5. (7 points) Solve $x^2 \frac{dy}{dx} + 2xy = 5y^3$. Express your answer in the form $y = y(x)$. Check your answer. **Your work must be coherent and meaningful.**

6. (7 points) Solve $y\frac{dy}{dx} + x = \sqrt{y^2 + x^2}$. Express your answer in the form $y = y(x)$. Check your answer. **Your work must be coherent and meaningful.**

7. (7 points) Consider two tanks. The first tank has a volume of 100 gals. of brine. The second tank has a volume of 200 gals. of brine. Each tank initially contains 50 lbs. of salt. Pure water flows into the first tank at the rate of 5 gal./min. The well mixed solution flows out of tank 1 and into tank 2 at the rate of 5 gal./min. The well mixed solution flows out of tank 2 at the rate of 5 gal./min. **Your work must be coherent and meaningful.**
   (a) How much salt is in the first tank at time t ?
   (b) How much salt is in the second tank at time t ?