PRINT Your Name:_____

Quiz for January 14, 2010

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.**

Suppose the velocity v of a motorboat coasting in water satisfies the differential equation $dv/dt = kv^2$. The initial speed of the motorboat is 10 m/sec. and v is decreasing at the rate of 1 m/sec^2 when $v = 5$ m/s. What is the velocity of the boat at each time? Sketch a graph of v as a function of t.

ANSWER: We can find k before right away. On the one hand, $dv/dt = kv^2$. On the other hand, when $v = 5$, $dv/dt = -1$. So $-1 = (dv/dt)|_{v=5} = k(5)^2$. We see that $-1/25 = k$. Now we solve the differential equation: $dv/dt = (-1/25)v^2$. Separate the variables $dv/v^2 = -(1/25)dt$. Integrate both sides: $-1/v = -(1/25)t + C$. Plug in $t = 0$ to learn: $-1/10 = C$. We have found $-1/v = -(1/25)t - (1/10)$. Multiply both sides by -50 to get $50/v = 2t + 5$ or $\boxed{\dfrac{50}{2t+5} = v(t)}$. The graph has a vertical asymptote at $t = -5/2$ and a horizontal asymptote of $v = 0$: