Problem 9 in Section 7.3. Find the inverse Laplace transform of $F(s) = \frac{3s+5}{s^2-6s+25}$.

Solution. The denominator $s^2 - 6s + 25$ does not have real roots (because $b^2 - 4ac = 36 - 100$ is negative); so we will complete the square, rather than factor:

$$s^2 - 6s + 25 = (s^2 - 6s + 9) + 16 = (s-3)^2 + 16.$$

We see that

$$\begin{aligned}\mathcal{L}^{-1}\left(\frac{3s+5}{s^2-6s+25}\right) &= \mathcal{L}^{-1}\left(\frac{3s+5}{(s-3)^2+16}\right)\\ &= \mathcal{L}^{-1}\left(\frac{3(s-3)+5+9}{(s-3)^2+16}\right)\\ &= 3\mathcal{L}^{-1}\left(\frac{(s-3)}{(s-3)^2+16}\right) + \frac{14}{4}\mathcal{L}^{-1}\left(\frac{4}{(s-3)^2+16}\right)\end{aligned}$$

We use $\mathcal{L}^{-1}F(s) = f(t) \implies \mathcal{L}^{-1}(F(s-a)) = e^{at}f(t)$.

$$= \boxed{3e^{3t}\cos 4t + \tfrac{7}{2}e^{3t}\sin 4t}$$