Problem 13 in Section 7.3. Find the inverse Laplace transform of $F(s) = \frac{5-2s}{s^2+7s+10}$.

Solution. We notice that $s^2 + 7s + 10 = (s+2)(s+5)$. We apply the technique of partial fractions to see

$$\frac{5-2s}{s^2+7s+10} = \frac{A}{s+2} + \frac{B}{s+5}.$$

Multiply both sides by (s+2)(s+5)

$$5 - 2s = A(s+5) + B(s+2).$$

Plug in s = -2 to learn that A = 3. Plug in s = -5 to learn B = -5. Observe that

$$\frac{5-2s}{s^2+7s+10} = \frac{3}{s+2} + \frac{-5}{s+5}.$$

We compute

$$\mathcal{L}^{-1}\left(\frac{5-2s}{s^2+7s+10}\right) = \mathcal{L}^{-1}\left(\frac{3}{s+2} + \frac{-5}{s+5}\right) = \boxed{3e^{-2t} - 5e^{-5t}}.$$