

Problem 1 in Section 7.3. Find the Laplace transform of $f(t) = t^4 e^{\pi t}$.

Solution. We use: if $\mathcal{L}(f(t)) = F(s)$, then $\mathcal{L}(e^{at} f(t)) = F(s - a)$. We also use $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$. So

$$\mathcal{L}(t^4) = \frac{4!}{s^5}$$

and

$$\mathcal{L}(e^{\pi t} t^4) = \boxed{\frac{4!}{(s - \pi)^5}}.$$