Problem 1 in Section 7.2. Use Laplace transforms to solve the Initial Value Problem

$$x'' + 4x = 0, \quad x(0) = 5, \quad x'(0) = 0.$$

Solution. Let $X = \mathcal{L}(x)$. It follows that

$$\mathcal{L}(x') = s\mathcal{L}(x) - x(0) = sX - 5$$
$$\mathcal{L}(x'') = s\mathcal{L}(x') - x'(0) = s(sX - 5) - 0 = s^2X - 5s$$

Apply $\mathcal{L}$ to $x'' + 4x = 0$ to obtain

$$s^2X - 5s + 4X = 0$$

$$X(s^2 + 4) = 5s$$

$$X = \frac{5s}{s^2+4}$$

$$x = \mathcal{L}^{-1}(X) = \mathcal{L}^{-1}\left(\frac{5s}{s^2+4}\right) = 5\mathcal{L}^{-1}\left(\frac{s}{s^2+4}\right) = 5\cos(2t).$$

$$\boxed{x = 5\cos 2t}.$$

Check. Plug

$$x = \cos 2t$$
$$x' = -2\sin 2t$$
$$x'' = -4\cos 2t$$

into $x'' + 4x$ and obtain $-4\cos 2t + 4\cos 2t = 0\checkmark$; $x(0) = 1\checkmark$; and $x'(0) = 0.\checkmark$ Our proposed answer does everything it is supposed to do. It is correct.