Problem 3 in Section 7.1. Use the definition of $\mathcal{L}$ to compute $\mathcal{L}(f(t))$ for $f(t) = e^{3t+1}$.

Solution. Recall that $\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\, dt$. We compute

$$\begin{aligned}
\mathcal{L}(t) &= \int_0^\infty e^{-st} e^{3t+1}\, dt \\
&= \int_0^\infty e^{(3-s)t+1}\, dt \\
&= \lim_{b\to\infty} \frac{1}{3-s} e^{(3-s)t+1} \Big|_0^b \\
&= \lim_{b\to\infty} \frac{1}{3-s} e^{(3-s)b+1} - \frac{e}{3-s}
\end{aligned}$$

If s is a constant with $3 < s$, then $3 - s$ is negative and $\lim_{b\to\infty} e^{(3-s)b+1} = 0$.

$$= \boxed{\frac{e}{s-3}}, \quad \text{provided } 3 < s.$$