Problem 25 in Section 7.1. Find the inverse Laplace transform of $F(s) = \frac{1}{s} - \frac{2}{s^{5/2}}$.

Solution. The fact sheet tells us that $\mathcal{L}(1) = \frac{1}{s}$ and $\mathcal{L}(t^a) = \frac{\Gamma(a+1)}{s^{a+1}}$. The fact sheet also tells us that $\Gamma(x+1) = x\Gamma(x)$ and $\Gamma(\frac{1}{2}) = \sqrt{\pi}$. Thus, when $a = \frac{3}{2}$,

$$\mathcal{L}(t^{\frac{3}{2}}) = \frac{\Gamma(\frac{3}{2}+1)}{s^{\frac{3}{2}+1}} = \frac{\frac{3}{2}\Gamma(\frac{3}{2})}{s^{\frac{5}{2}}} = \frac{\frac{3}{2}\Gamma(1+\frac{1}{2})}{s^{\frac{5}{2}}} = \frac{\frac{3}{2}(\frac{1}{2})\Gamma(\frac{1}{2})}{s^{\frac{5}{2}}} = \frac{3\sqrt{\pi}}{4s^{\frac{5}{2}}}.$$

We conclude that

$$\begin{aligned} \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{2}{s^{5/2}}\right) &= \mathcal{L}^{-1}\left(\frac{1}{s}\right) - 2\mathcal{L}^{-1}\left(\frac{1}{s^{\frac{5}{2}}}\right) \\ &= \boxed{1 - 2\left(\frac{4}{3\sqrt{\pi}}t^{\frac{3}{2}}\right)}. \end{aligned}$$