Problem 17 in Section 7.1. Compute $\mathcal{L}(f(t))$ for $f(t) = \cos^2 2t$.

Solution. The Laplace transform fact sheet does not directly show how to do this problem. Maybe we can rewrite the problem in a different form and use the fact sheet on the new form. In calculus if you had to integrate $\cos^2 x$, you used the identity that

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x). \tag{20}$$

Of course, that identity will help us here. If you don't immediately know (20), maybe you remember

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi. \tag{21}$$

(Of course (21) is an immdeiate consequence of Euler's identity

$$e^{i\theta} = \cos(\theta) + i\sin(\theta).$$

Just expand both sides of

$$e^{i\theta}e^{i\phi} = e^{i(\theta+\phi)}$$

and equate the real part of each side.) At any rate, if you know (21), then

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x);$$

hence

$$\cos 2x = 2\cos^2 x - 1$$

and therefore (20).

As soon as you know (20), then the problem is easy:

$$\mathcal{L}(\cos^2 2t) = \tfrac{1}{2}\mathcal{L}(1 + \cos 4t) = \boxed{\tfrac{1}{2}(\tfrac{1}{s} + \tfrac{s}{s^2+16})}.$$

Of course, the fact sheet tells us that $\mathcal{L}(1) = \frac{1}{s}$ and $\mathcal{L}(\cos kt) = \frac{s}{s^2+k^2}$.