**Problem 3 in Section 3.3.** Find the general solution of $y'' + 3y' - 10y = 0$.

**Solution.** We try $y = e^{rx}$. We plug $y$, $y' = re^{rx}$ and $y'' = r^2e^{rx}$ into the Differential Equation. We want

$$r^2e^{rx} + 3re^{rx} - 10e^{rx} = 0.$$

We want $e^{rx}(r^2 + 3r - 10) = 0$. If a product is zero, one of the factors must be zero. The function $e^{rx}$ is never zero; so we want $r^2 + 3r - 10 = 0$. We want $(r + 5)(r - 2) = 0$ In other words, $r = 2$ or $r = -5$. The general solution of $y'' + 3y' - 10y = 0$ is $\boxed{y = c_1e^{2x} + c_2e^{-5x}}$.

**Check.** We plug

$$\begin{aligned} y &= c_1e^{2x} + c_2e^{-5x} \\ y' &= 2c_1e^{2x} - 5c_2e^{-5x} \\ y'' &= 4c_1e^{2x} + 25c_2e^{-5x} \end{aligned}$$

into $y'' + 3y' - 10y$ and obtain

$$\begin{aligned} &\left(4c_1e^{2x} + 25c_2e^{-5x}\right) + 3\left(2c_1e^{2x} - 5c_2e^{-5x}\right) - 10\left(c_1e^{2x} + c_2e^{-5x}\right) \\ &= (4c_1 + 6c_1 - 10c_1)e^{2x} + (25c_2 - 15c_2 - 10c_2)e^{-2x} = 0.\checkmark \end{aligned}$$