

Problem 3 in Section 3.2. Find a nontrivial linear combination of $f(x) = 0$, $g(x) = \sin x$, and $h(x) = e^x$ which is the constant function zero.

Solution. We want numbers (at least one not zero) a_1, a_2, a_3 , with $a_1f(x) + a_2g(x) + a_3h(x)$ equal to the constant function 0. This is easy. Let a_1 be any nonzero number and $a_2 = a_3 = 0$. For example

$$\boxed{\frac{75}{43}f(x) + 0g(x) + 0h(x)}$$

is the constant function 0.