

**Problem 26 in Section 3.2.** Find a particular solution for each of the following Differential Equations.

(a)  $y'' + 2y = 4$ ,

(b)  $y'' + 2y = 6x$ ,

(c)  $y'' + 2y = 6x + 4$ .

We learn how to find particular solutions of non-homogeneous linear Differential Equations with constant coefficients in section 3.5. The basic technique is, “Guess the form of the answer and then adjust the coefficients”. This problem serves as a warm-up for the procedure.

Solution. If we want to plug some function into  $y'' + y$  and end up with a constant; we should probably plug a constant in. Which constant should we plug in. Either you blurt the answer out or you think “If  $y = C$  and  $y'' + 2y = C$ , then  $0 + 2C = 4$ ; so we should take  $C = 2$ . Sure enough,  $y = 2$  is a solution of  $y'' + 2y = 4$ .

Similarly, we try  $y = Ax + B$  as a candidate for a solution for  $y'' + 2y = 6x$  and calculate  $y' = A$  and  $y'' = 0$ . If  $y = Ax + B$  is a solution of  $y'' + 2y = 6x$ , then  $0 + 2(Ax + B) = 6x$ . We better have  $2A = 6$  and  $B = 0$ . Thus,  $y = 3x$  is a solution of  $y'' + 2y = 6x$ .

For (c) you could either combine the answers to (a) and (b) or start from scratch with  $y = Ax + B$ . At any rate  $y = 3x + 2$  is a solution of  $y'' + 2y = 6x + 4$ .