

Problem 22 in Section 3.1. Are the functions $f(x) = 1 + x$ and $g(x) = 1 + |x|$ linearly independent or linearly dependent?

Solution.

The functions f and g are linearly independent.

There is no number α with $f(x) = \alpha g(x)$ for all x . (It is clear that

$$f(x) = 1g(x), \quad \text{when } x \text{ is non-negative;}$$

but $f(x) \neq 1g(x)$ when x is negative. In fact, there is no constant α with $f(x) = \alpha g(x)$ for all negative numbers x .)