

Problem 20 in Section 3.1. Are the functions $f(x) = \pi$ and $g(x) = \cos^2 x + \sin^2 x$ linearly independent or linearly dependent?

Solution This problem is tricky. Actually both functions are constant. Indeed $g(x) = 1$ for all x . So, $f(x)$ is a constant multiple of $g(x)$. That is $f(x) = \pi g(x)$ for all x . We conclude that

$f(x)$ and $g(x)$ are linearly dependent.

The point is: The general solution of a second order homogeneous linear Differential Equation has the form $c_1 y_1 + c_2 y_2$ where y_1 and y_2 are linearly independent (i.e., really different) solutions of the Differential Equation and c_1 and c_2 are arbitrary constants. So, $c_1 \pi + c_2 (\cos^2 x + \sin^2 x)$ is not the general solution of any second order homogeneous linear Differential Equation.