Problem 17 in Section 3.1. Show that $y = \frac{1}{x}$ is a solution of $y' + y^2 = 0$, but that $y = \frac{c}{x}$ is not a solution of $y' + y^2 = 0$, unless c happens to be zero or one.

The point of this problem is that our tricks for linear Differential Equations do not work for non-linear Differential Equations. In particular if y_1 is a solution of a homogeneous linear Differential Equation, then cy_1 is also a solution of the Differential Equation. This statement is not true for non-linear Differential Equations.

Solution Plug $y = \frac{1}{x}$ and $y' = \frac{-1}{x^2}$ into $y' + y^2$ and get $\frac{-1}{x^2} + (\frac{1}{x})^2$, which is zero.

On the other hand, we we plug $y = \frac{c}{x}$ and $y' = \frac{-c}{x^2}$ into $y' + y^2$ we obtain $\frac{-c}{x^2} + (\frac{c}{x})^2 = \frac{c^2-c}{x^2}$. The function $\frac{c^2-c}{x^2}$ is not the zero function unless $c(c-1)-0$; that is unless $c = 0$ or $c - 1 = 0$.